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## EXPERIMENTS ON FRUSTRATED JOSEPHSON ARRAYS

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The resistive behaviour of a square Josephson junction array as a function of frustration  $f$  has been studied in detail. At  $f = 1/2$  a very pronounced dip in resistance is found, other clear minima occur at  $f = 1/3$ ,  $1/5$  and  $1/10$ . For small values of  $f$  below 0.08, the resistance as a function of temperature shows thermally activated behaviour, for  $f > 0.08$  this is not the case. At  $f = 1/2$  the nonlinear resistance shows a temperature dependence which contains aspects of a Kosterlitz–Thouless transition.

### 1. Introduction

On one hand, two-dimensional arrays of superconducting islands, weakly connected by Josephson junctions, form a controlled model system for the study of some aspects of granular films, which can be viewed as a disordered network of Josephson junctions. From a more fundamental point of view, the Josephson junction array is a physical representation of the 2D  $XY$  model, an intensive area of study for the theory of phase transitions, both in the classical and the quantum regime. In the latter, because of quantum phase fluctuations, no transition to an ordered state is expected [1]. However, in the classical regime, thermal excitations in the form of logarithmically interacting vortex-antivortex pairs can exist without destroying long range order. From this state, the transition to a disordered phase is described by the Kosterlitz–Thouless (KT) theory [2] and its extensions [3–5].

In the theoretical description of the  $XY$  model, the coupling energy between two sites of the lattice is taken as a constant. In Josephson tunnel junction arrays, this energy is proportional to the temperature-dependent junction critical current. Therefore, a dimensionless temperature  $\tau$  can be defined as the quotient of the thermal and Josephson coupling energy:

$$\tau = \frac{k_B T}{(\hbar/2e)i_c(T)}, \quad (1)$$

where  $i_c(T)$  is the critical current per junction.

When a magnetic field is applied perpendicular to the plane of the islands and junctions, the array is equivalent to the frustrated  $XY$  model. The frustration  $f$  is given by the flux through one unit cell of the array, normalized to the flux quantum  $\phi_0 = h/2e$ . Of special interest is the fully frustrated case  $f = 1/2$  [6–10]. The ground state is twofold degenerated, leading to formation of domain walls. Monte Carlo simulations seem to indicate that a KT like transition coincides with an Ising one [9]. A proposed mechanism [10] is the unbinding of fractional vortices, placed at the corners of domain walls, in a similar way as in the KT transition with integer vortices.

Here we will present results on the resistive behaviour of junction arrays at small values of  $f$  and at  $f = 1/2$ . Near  $f = 0$  in both the linear and the nonlinear resistance, the measurements agree with the predictions of the KT theory. At small values of  $f$  ( $f < 0.08$ ) thermally activated behaviour is found in the linear resistance. At  $f = 1/2$  in the linear and the nonlinear resistance KT aspects are seen. The transition temperature does not correspond to the theoretical value.

### 2. Fabrication and experimental set-up

The junctions are niobium oxide tunnel junctions, fabricated using a shadow evaporation

method. The junctions measure  $1 \times 0.2 \mu\text{m}^2$ . By varying the oxide thickness, a KT transition temperature of about 5 K is chosen. Typical values of the normal state resistance per junction and the junction critical current at the transition temperature are  $300 \Omega$  and  $200 \text{ nA}$ , respectively.

The unit cell in the array has an area of  $50 \mu\text{m}^2$ , leading to a perpendicular field of 0.4 G for one flux quantum in each unit cell. The sample room is shielded by superconducting lead and by  $\mu$ -metal. All measurements were performed in a standard helium-4 cryostat. The voltage measurements were obtained using a lock-in technique with a frequency of 108 Hz and a current of  $2 \text{ nA}$  per junction for the linear resistances. The experimental resolution, limited by noise, was about  $10^{-9} \text{ V}$ .

At different temperatures, the critical junction current is extracted straightforwardly from the array voltage-current characteristics. Knowing  $i_c(T)$ , the normalized temperature  $\tau$  is used as the relevant temperature scale in all plots and analyses.

### 3. Magnetoresistance

Several arrays of different widths have been fabricated. Ladders (one unit cell wide,  $N=1$ ) are studied as well as arrays with width  $N=2, 3, 7, 15, 127$ . In all these arrays periodic oscillations with one flux quantum per unit cell are found. In fig. 1 the resistance of an array of width 127 for several temperatures is given. At the KT transition temperature and lower, the oscillations with  $f=1$  are clearly present. In the most homogeneous samples, these oscillations persist up to  $f=100$ . With more disorder the amplitudes decrease more rapidly. Above this fundamental oscillation of  $f=1$ , another oscillation with a period of about 30 is found; in some arrays even a smaller period of  $f=10$ . Chains of junctions, with closely similar parameters, show the same oscillations, indicating individual island/junction behaviour. The mechanism itself is not fully understood at present. In fig. 2 an example is given of an array with  $N=127$ . Both the critical current and the resistance are plotted in arbitrary

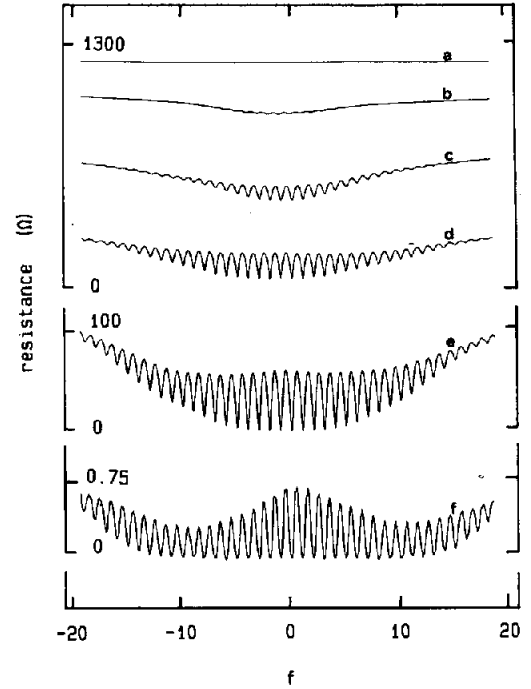


Fig. 1. The resistance of an array of width 127 (sample #115) as a function of frustration  $f$  for several normalized temperatures: a:  $\tau = 100$ ; b:  $\tau = 10$ ; c:  $\tau = 2.45$ ; d:  $\tau = 1.0$  e:  $\tau = 0.73$ ; f:  $\tau = 0.30$ .

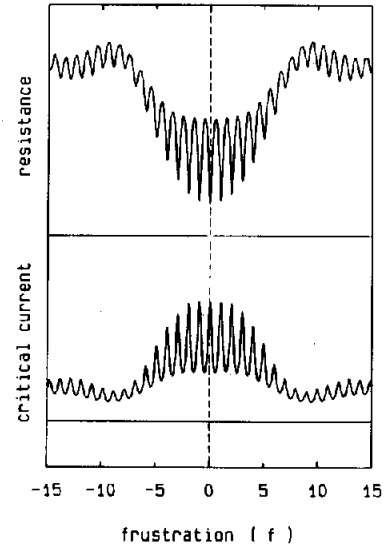


Fig. 2. The resistance (upper part) and the critical current (lower part) as a function of frustration  $f$ .

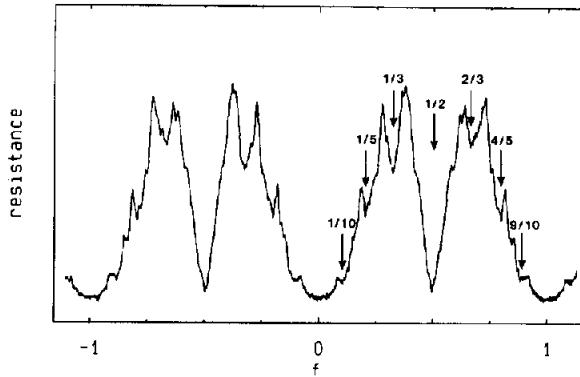


Fig. 3. The resistance of an array of width 15 as a function of frustration  $f$ . The maximum resistance is  $1/1000$  of the normal state resistance.

units. Solid lines indicate zero resistance and critical current.

Observation of structure at fractional values of  $f(p/q)$  is seen at temperatures well below the KT transition temperature. This is in accordance with the fact that phase coherence over  $q$  lattice spaces is required, instead of 1 as for the  $f=1$  oscillations. In general, in all arrays, the first non-integer resistance dip is seen at  $f=1/2$ . By lowering the temperature, more structure at larger values of  $q$  become visible. At the lowest temperatures even structure for  $q=10$  has been observed. In fig. 3 a magnetoresistance curve for an array with  $N=15$  is given. At  $f=0$  the resistance is zero within measuring accuracy. Even for  $f=1/2$  the resistance comes down to about  $1/50$  of the maximum resistance in the plot. Other clear dips occur at  $f=1/3, 2/3, 1/5, 4/5, 1/10$  and  $9/10$ , which are denoted in the figure. Note that structure is also present for  $q=4, 6, 8$ , although surprisingly little pronounced.

In the remainder, the resistive behaviour of one array of width 127 and length 383 (fig. 1) will be discussed in detail. That particular array showed very pronounced dips for  $f=1/2, 1/3$  and oscillations with  $f=1$  for more than 100 periods.

#### 4. Resistive behaviour at $f=0$

As function of temperature, the resistance

shows a very broad transition, going down roughly from the normal state value at 7.5 K to zero resistance at about 5 K. Near the KT transition temperature, the resistance is predicted to follow a square root cusp [3, 5]:

$$\frac{R}{R_N} = a \exp\left\{\frac{-b}{(\tau - \tau_c)^{0.5}}\right\}, \quad (2)$$

where  $b$  is a constant of order unity. In fig. 4 the dashed line follows formula 2, with parameters  $a=1.6$ ,  $b=2$  and  $\tau_c=0.98$ , in good agreement with theory.

In the low resistance regime, the data deviate from the predicted line. Plotted as a function of  $1/\tau$ , exponential decrease is found. The related energy barrier in terms of the Josephson coupling energy ( $E_J$ ) is equal to about 6, much too small to be straightforwardly due to vortex crossing over the finite array in zero field ( $\pi \ln(N/2)$ ). The minimum in the resistance does not exactly correspond to zero current in the Helmholtz coils, used to apply a certain amount of frustration. Our zero is shifted by about 3 mG and correspondingly adjusted. However, it is very likely that this residual field is inhomogeneous and not exactly compensated by the homogeneous Helmholtz field. We estimate that the inhomogeneous part is of order 0.4 mG, well above the critical field  $H_{c1}$ . Consequently, in certain parts of the array magnetic field induced vortices will be present, even at  $f=0$ .

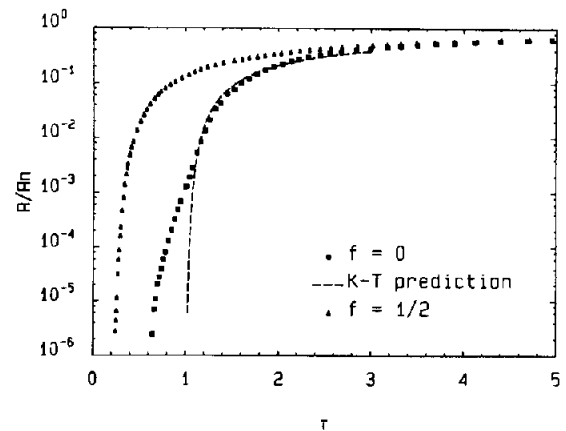


Fig. 4. The linear resistance of sample #115 as a function of the normalized temperature for  $f=0$  and  $f=1/2$ . The dashed line is a fit to the KT prediction of formula (2).

As a function of  $\tau$  the resistance is shown in fig. 5. The transition plotted in this way shows aspects similar to the fluctuation model of Aslamazov and Larkin [11]. Halperin and Nelson (HN) [3] gave an interpolated expression, applicable for all temperatures above  $\tau_c$  and incorporating Aslamazov–Larkin like fluctuations. Their formula, translated to resistances and normalized temperatures,

$$\frac{R}{R_N} = \frac{1}{1 + a \sinh^2 \left\{ \frac{b}{(\tau - \tau_c)^{0.5}} \right\}}, \quad (3)$$

fits the data of fig. 5 remarkably well. The fitting parameters are  $a = 1.3$ ,  $b = 1.2$  and  $\tau_c = 1.0$ .

Another feature of the data is that they can reasonably well be fitted to the single-junction Ambegoakar-Halperin form, with an energy barrier of  $7.6E_J$ . This value of 7.6 could indicate fluctuations of independent islands, requiring about 4 times the single-junction barrier ( $2E_J$ ) for a phase slip of  $2\pi$ .

In the low current region, the  $I$ - $V$  characteristics are linear, even for temperatures below the KT transition temperature. This linear behaviour is believed to be caused by the presence of the free magnetic vortices, caused by the ambient magnetic field. Recently a similar result has been reported for thin-film superconducting films [12]. In the high current region nonlinearity in the

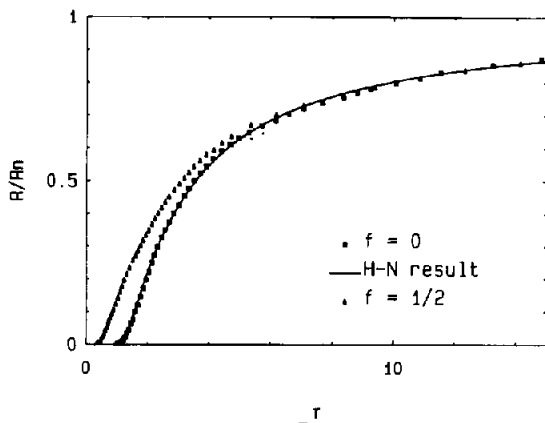


Fig. 5. The linear resistance of sample #115 as a function of the normalized temperature for  $f = 0$  and  $f = 1/2$ . The solid line through the data of  $f = 0$  is a fit of formula (3).

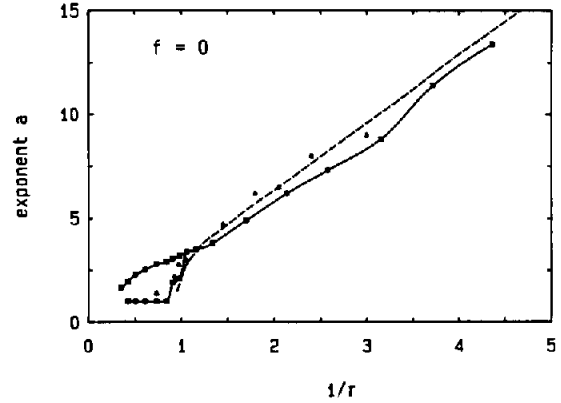


Fig. 6. Exponent  $a$  of the non-linear resistance as a function of inverse normalized temperature for  $f = 0$ . Squares indicate the results of ref. [13]. The upper line of data is extracted at  $V = 10^{-5}$  V, the lower at  $V = 3 \times 10^{-9}$  V. Triangles are the data of sample #115. The dashed line is a Monte Carlo simulation (ref. [6]).

form of power laws is present, yielding to  $V \sim I^{a(\tau)}$ . Fig. 6 shows our old data of ref. [13] together with the new results and predictions of Monte Carlo simulations. For the new results only the data extracted at  $10^{-5}$  V are given. The data of the two samples are in good agreement with each other and with the Monte Carlo simulations. The jump takes place at the same value of  $\tau$  as found in the fit of the square root cusp behaviour of formula 2. The vortex-antivortex unbinding mechanism establishes itself in both the linear and in the nonlinear resistance, leading to the same transition temperature. For very small values of  $f$ , one can conclude that the KT behaviour is clearly present and that the data in the higher resistance region fit the theoretical KT predictions very well.

## 5. Resistive behaviour for $f < 0.08$

For all values of  $f$  smaller than 0.08 exponential decrease with inverse normalized temperature is found at lowest temperatures (fig. 7). The dependence can be expressed as

$$\ln \frac{R}{R_N} = -\alpha \tau^{-1}, \quad (4)$$

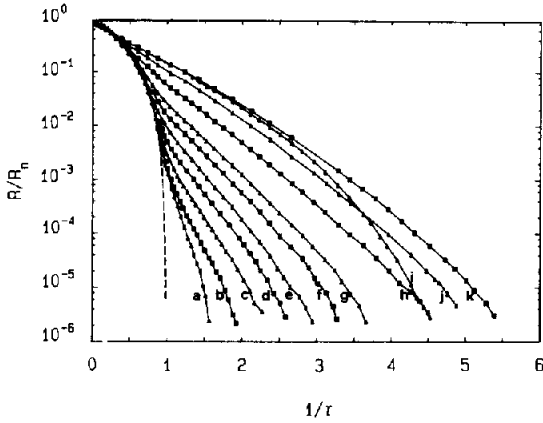


Fig. 7. The linear resistance of array #115 as a function of inverse normalized temperature for several values of the frustration  $f$ : a:  $f=0$ ; b:  $f=0.0008$ ; c:  $f=0.002$ ; d:  $f=0.0038$ ; e:  $f=0.0068$ ; f:  $f=0.0135$ ; g:  $f=0.025$ ; h:  $f=0.078$ ; i:  $f=0.5$ ; j:  $f=0.193$ ; k:  $f=0.401$ .

where  $\alpha$  is equal to the slope of the curves times  $E_J$ . Exponential decrease as a function of temperature is also found in thin films, recently explained in terms of a magnetic field and temperature dependent vortex pinning [12]. In our case the energy barriers are almost temperature independent and also seem to be sample independent. Furthermore the barriers involved are rather high, in the order of 4 to 6 times  $E_J$ . Below  $H_{c1}$ , we proposed a model in which vortex crossing is the dominant mechanism for the resistance with a magnetic field dependent barrier [14]. A natural extension of this model for small values of  $f$  could be as follows: still the crossing of vortices is the most important mechanism for resistance, but with the modification of interaction with stationary vortices induced by the ambient field. It seems very difficult to calculate the effective energy barrier for crossing of thermally excited vortices. As  $f$  becomes too large the resistance will be dominated more and more by the magnetic vortices, resulting in a different temperature dependence. Indeed such a qualitative picture is shown by fig. 7.

In order to verify this concept, we have fabricated Josephson junction arrays in a corbino geometry: a radial array of width 127, with on the inside as well as the outside superconducting tracks at the boundaries. Flux quantization is

avoided by leaving free a space of 8 junctions. On the same substrate a usual array was placed. By the surrounding of the superconducting tracks, one expects in the corbino array to observe flux flow without finite size effects. Experimentally, for several values of  $f$ , including  $f=0$  and  $1/2$ , only slight differences were observed in the resistance.

## 6. Resistive behaviour for $f=1/2$

In fig. 4, the linear resistance is given as a function of  $\tau$ . If one uses the KT expression for the resistive behaviour (formula (2), a reasonable fit is obtained with a transition temperature of 0.2. In fig. 6, the curve i ( $f=1/2$ ) behaves differently than the other curves. No exponential decrease and more curvature than at  $f=0.4$  is found. At the lowest temperatures, the resistance is even lower than for  $f=0.078$ , and about 20 times smaller than the maximum resistance.

On the same array, the nonlinear resistance measurements for  $f=1/2$  are performed. The behaviour is quite similar to that at  $f=0$ . Below some temperature, straight lines in the  $\log V - \log I$  plot are found over nearly 5 orders of magnitude of  $V$ . Above this temperature, the lines start to curve. As a function of the voltage the exponent ( $\alpha$ ) can be extracted from the  $V-I$  plot. Fig. 8 gives the data for  $f=1/2$  for two values of  $V$  ( $10^{-5}$  and  $3 \times 10^{-9}$  V). Different voltages and currents mean probing the system at different length scales. As  $V$  decreases the more the exponent is expected to jump. Although the jump is not as steep as in the  $f=0$  case, clearly certain KT aspects are present.

In fig. 8 we also reproduce earlier results on the exponent  $\alpha$  as a function of  $1/\tau$ , as published in ref. [13]. In fig. 6 the data for  $f=0$  are also indicated for that array. Clearly, at  $f=0$  good agreement exists between earlier and new results. However, at  $f=1/2$  a significant shift in  $\tau_c$  is observed, the earlier results showing the higher transition temperature. The Monte Carlo simulations indicate a transition at even higher temperature. The main difference between the

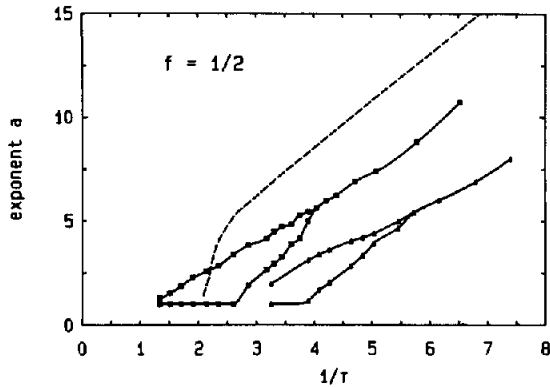


Fig. 8. Exponent  $a$  of the non-linear resistance as a function of inverse normalized temperature for  $f = 1/2$ . Squares indicate the results of ref. [13] and triangles the ones for sample #115. The upper lines of data are extracted at  $V = 10^{-5}$  V, the lower ones at  $V = 3 \times 10^{-9}$  V. The dashed line is a Monte Carlo simulation (ref. [6]).

old and new arrays we think is the amount of disorder, due to a recent improvement in fabrication procedure (e.g. indicated by lower  $f = 1/2$  minimum in magnetoresistance). Comparison of figs. 6 and 8 seems to indicate that a small amount of disorder has little influence at  $f = 0$ , but significant quantitative influence for the transition at  $f = 1/2$ . The transition temperature as determined from  $a(\tau)$  is the same as the one following from the linear resistance.

It is interesting to compare the previously mentioned results with the recent Monte Carlo simulations of Thijssen and Knops [15]. They varied the Ising coupling independently from the KT one. If the Ising coupling is weakened while leaving the KT coupling the same, the transition temperature becomes smaller and the jump in the helicity modulus becomes in a nonuniversal way higher. In our case the Ising interaction might be modified by small disorder which leaves the KT order on a somewhat larger scale undisturbed. The  $f = 0$  case would not be modified.

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